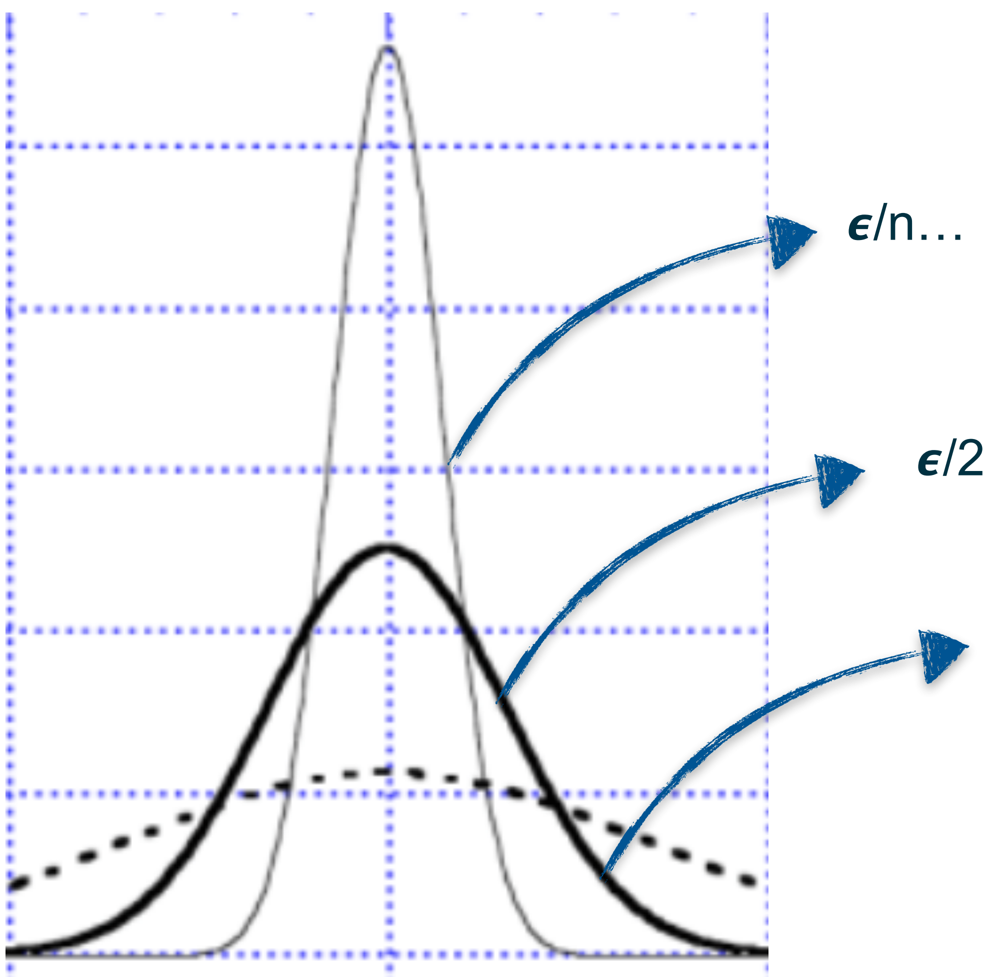
We are going to approximate Dirac delta functions as Gaussians… can we actually do that?

Well, you can express a Dirac delta function ***δ(x)*** as the limit of (a sequence of) functions - Gaussians, for example, of width $\boldsymbol{\epsilon}$



$$\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\sqrt{\epsilon\pi}} \cdot e^{-\frac{x^2}{\epsilon}}$$

Note that:

$$\forall \epsilon \neq 0, \quad \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\epsilon\pi}} \cdot e^{-\frac{x^2}{\epsilon}} dx = 1$$

Which, being independent of $\boldsymbol{\epsilon}$, holds even for $\boldsymbol{\epsilon} \to 0$