Electronic Structure Methods for Materials Modelling

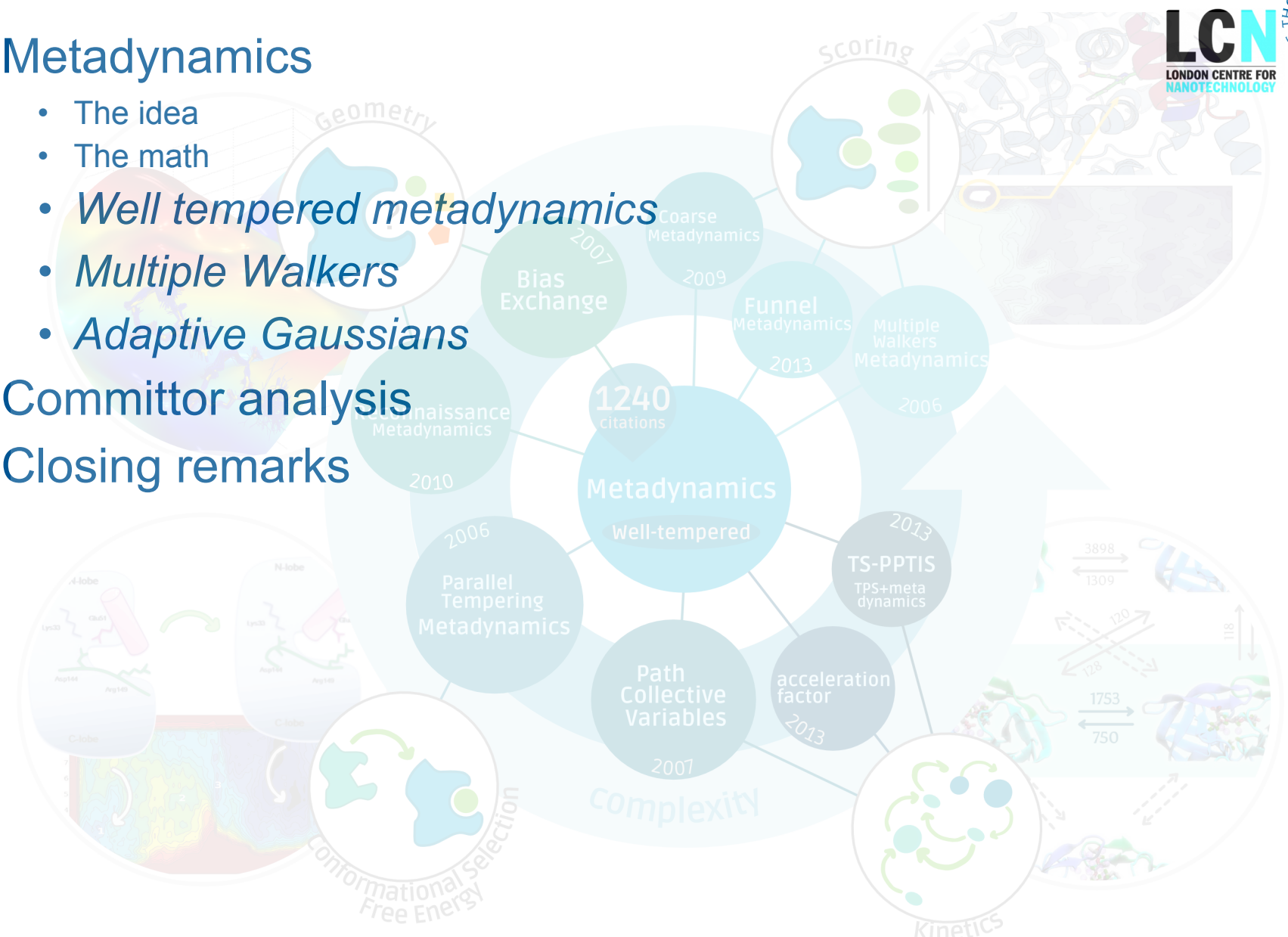
A peek into the wondrous realm of the

Enhanced Sampling (Vol.2)

- Learning Outcomes
  - Metadynamics: ***the concept of a history-dependent bias***
  - The key choice: ***why order parameters do matter***

Outline

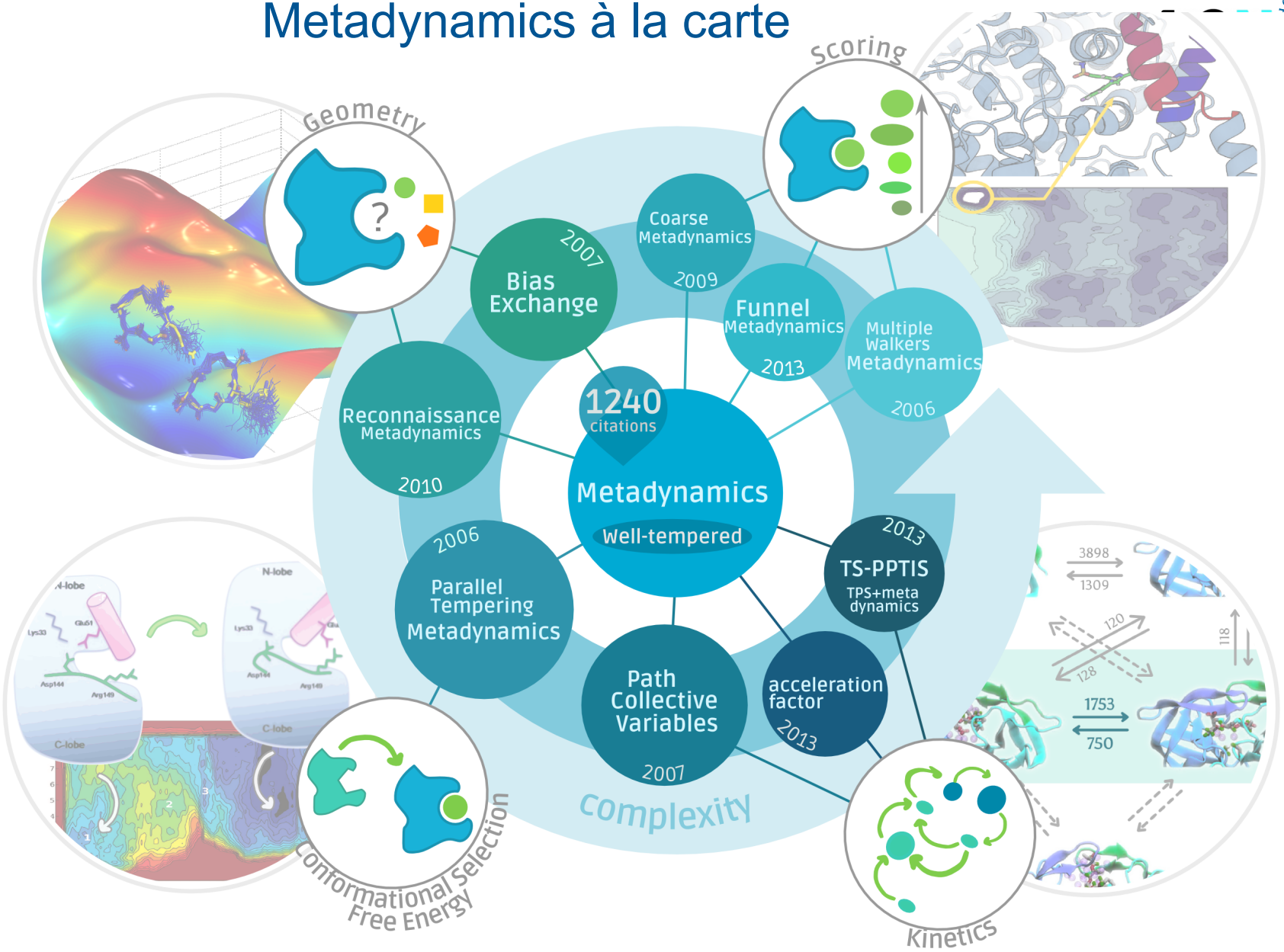
- Metadynamics
  - The idea
  - The math
  - *Well tempered metadynamics*
  - *Multiple Walkers*
  - *Adaptive Gaussians*
- Committor analysis
- Closing remarks



Next: Surfaces & Interfaces

Outline

Metadynamics à la carte



Metadynamics

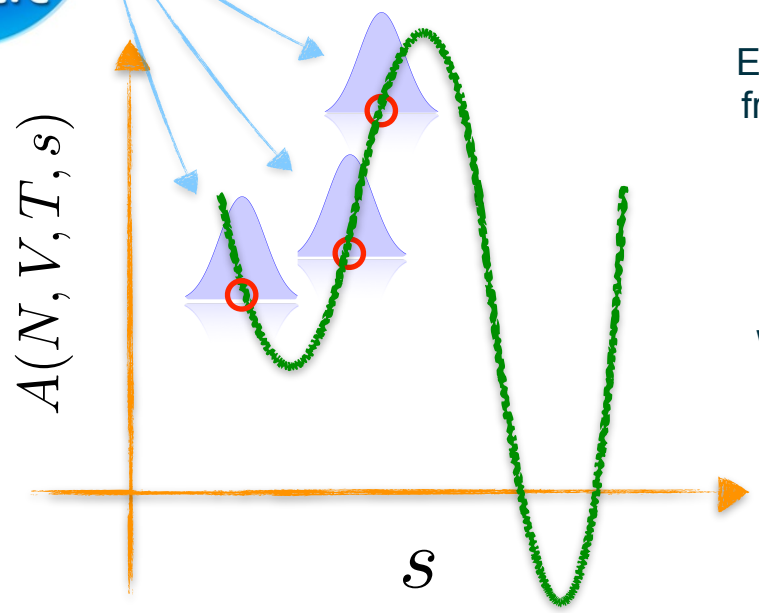
The idea

Up to know, we have been dealing with *static* bias:

- In Thermodynamic Integration or Blue Moon ensemble, we perform a series of MD runs for different values of our CV
- The same holds for Umbrella Sampling: the bias does not depend on time

Introducing a time-dependent bias, or - to be precise - an *history dependent* bias:
Many ways to do it! Adaptively biased MD, Local Elevation Umbrella Sampling, and…

Metadynamics
A very popular option within the AIMD community



The idea:
A single MD run
Every time we visit an already visited point of the free energy surface (FES), we add some bias (a Gaussian) in there.

We lower the probability of exploring the same region of the FES during the MD run

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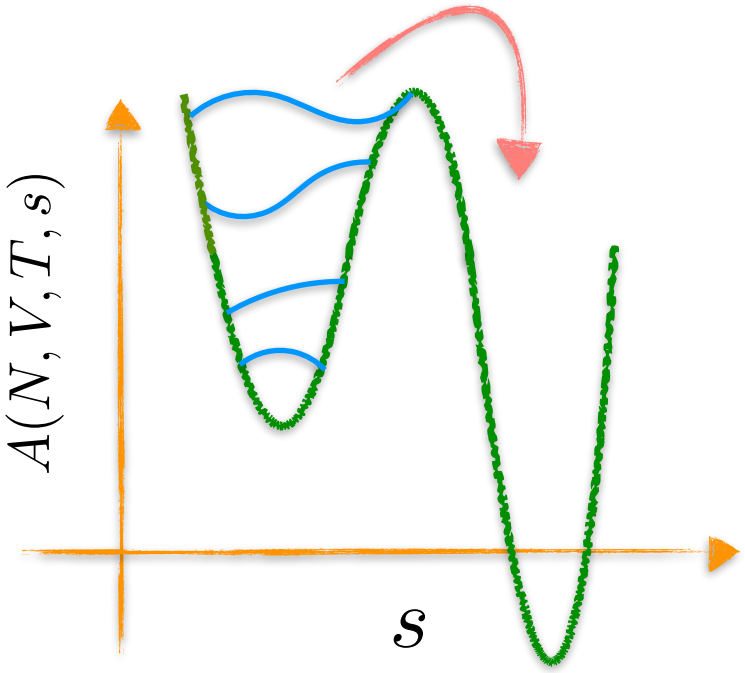


This way, we literally fill the FES till the system crosses the barrier, thus escaping toward another minimum

You can do that till you flatten the FES, obtaining the full free energy profile from the bias you have added

Note: in principle, you don't need any a priori information about the FES

Metadynamics

The - quick - math

Let us start again from the probability of the CV *s*:

$$P(s) = \frac{1}{Q(N,V,T,)} \int \delta(f(\{\mathbf{r}\}) - s) \cdot e^{-\beta \mathcal{H}(\{\mathbf{r},\mathbf{p}\})} d\mathbf{r} d\mathbf{p}$$

$$= \langle \delta(f(\{\mathbf{r}\}) - s) \rangle$$

If one assumes ergodicity…

$$P(s) = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau \delta(f(\{\mathbf{r},\mathbf{t}\}) - s) dt$$

Metadynamics

The - quick - math

The trick: replace the delta functions with Gaussians

$$P(s) = \lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau \delta(f(\{\mathbf{r},\mathbf{t}\}) - s)dt$$

$$\delta(f(\{\mathbf{r},\mathbf{t}\}) - s) = \lim_{\sigma\to 0} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(f(\{\mathbf{r},\mathbf{t}\})-s)^2}{2\sigma^2}}$$

In the limit of infinite height and zero width, this approximation is exact.
In practice, of course, this is not the case. The wider/higher the Gaussians…

$$\lim_{\sigma\to 0} \to \Delta s$$

In practice:

$$\lim_{\tau\to\infty} \to \sum_{k=0}^{N_t} (\Delta t)$$

This is in terms of the CV!
Has to be chosen!

$$P_{\Delta s}(s) = \frac{1}{t\Delta s\sqrt{2\pi}} \sum_{k=0}^{N-1} \exp -\frac{[f(\{\mathbf{r}(k\Delta t)\}) - s]^2}{2\Delta s^2}$$

Metadynamics

The - quick - math

So now you have a probability approximated as a sum of Gaussians over time:

$$P_{\Delta s}(s) = \frac{1}{t\Delta s\sqrt{2\pi}} \sum_{k=0}^{N-1} \exp - \frac{[f(\{\mathbf{r}(k\Delta t)\}) - s]^2}{2\Delta s^2}$$

But then, why not adding a time dependent bias *W* to our MD simulations as a sum of Gaussians over time?

$$W_G(s, \{\mathbf{r}(t)\}) = h_G \sum_{t'=\tau_G, t'<t}^{N\cdot\tau_G} \exp - \frac{[s\{\mathbf{r}\} - s(\{\mathbf{r_G}(t')\})]^2}{2\Delta s^2}$$

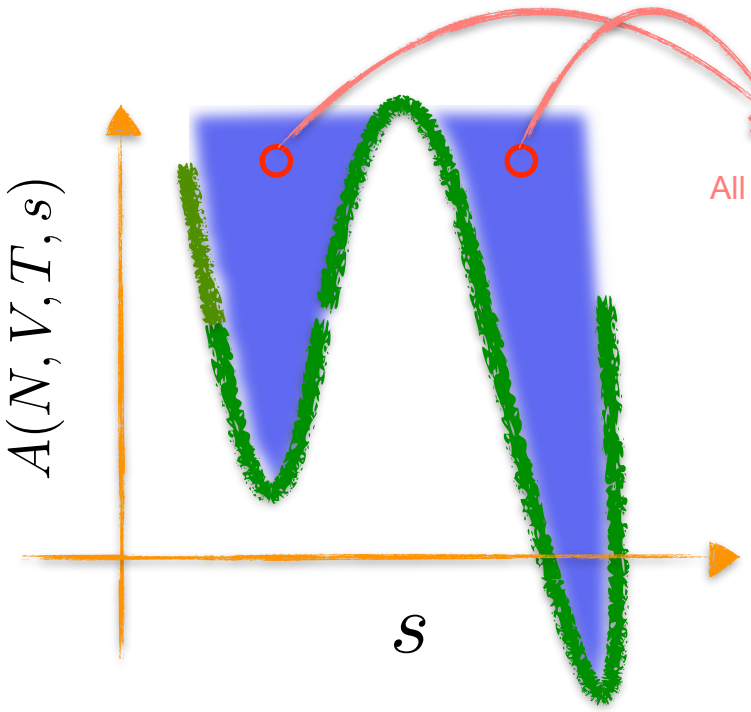
- h_G: height of each Gaussian (to be chosen)
- τ_G: defines how often (every how many MD steps) you add a Gaussian
- The sum over τ_G: keeps track of the whole history of the MD run
- Δs: width of each Gaussian (to be chosen)

A *point* of the FES which we have already visited

Metadynamics

The - quick - math

Let us assume you did things carefully, and that after a certain time t*
(in principle infinite…) you have actually "filled" the FES



All the bias *W* you have
piled up in time

It is rather intuitive that if this is the case, then

$$A(N,V,T,s) = -W_G(s,\{\mathbf{r}(t^*)\})$$

This equivalence has very recently been proven exact
(very tough derivation:)

Dama, James F., Michele Parrinello, and Gregory A. Voth 2014
Well-Tempered Metadynamics Converges Asymptotically. Physical Review Letters 112(24): 240602.

Well-Tempered Metadynamics

When

Metadynamics is rather useful not only for getting a FES, but also to explore it (new phases, new isomers, new reaction pathways…)

On the other hand, ***when do you stop***?
In a single metadynamics run, the FES does not converge to a definite value, but fluctuates around the actual results with an error:

$$\epsilon \propto \sqrt{\frac{S\ \Delta s\ h_G}{D\ \tau_G\ \beta}}$$

There is one way to make things smoother & well behaving:
Well-Tempered Metadynamics

The basic idea: the height h_G of the Gaussians we deposit decays with time (in standard metadynamics, h_G is constant)

Well-Tempered Metadynamics

The idea

We aim at rescaling h_G in such a way that:

- We explore exclusively the regions of the FES in which we are truly interested in
- In the long time limit, we converge to the exact FES
- We don't overfill - that is, we explore the FES as efficiently as possibile

To this end:

$$h_G(k\Delta t) = h_G(0) \cdot \exp\left[\frac{W(s(\{\mathbf{r}(k,\Delta t)\}, k\Delta t))}{k_B \Delta T}\right]$$

Initial height of the Gaussians

The bias at the current CV positions and at the current time step

Temperature-like parameter: it determines how quickly the height of the Gaussians decays in time

Within this approach, the free energy can be written as:

$$A(N,V,T,s) = -\frac{T+\Delta T}{\Delta T} \lim_{t\to\infty} W_G(s,\{\mathbf{r}(t)\})$$

Well-Tempered Metadynamics

A tremendous bargain

The central quantity in here is this temperature-like factor ΔT

Standard Molecular Dynamics

Standard Metadynamics

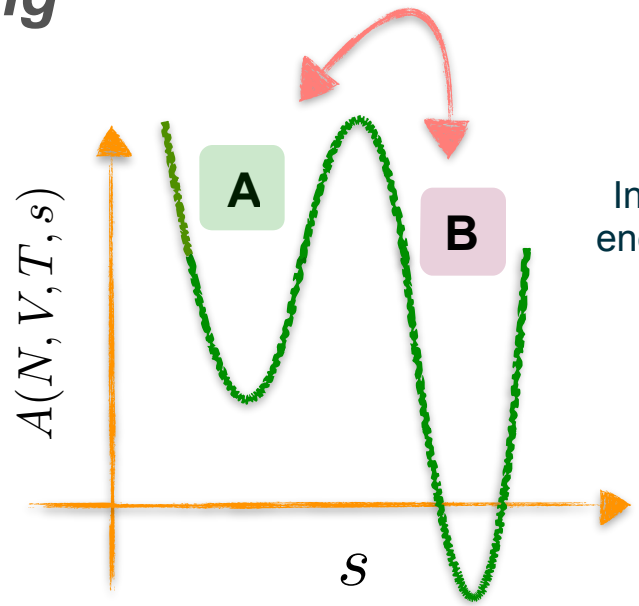
By tuning ΔT, one can:

- Facilitate the exploration in the CVs space
- Limit the exploration of the FES region to an energy range of the order $T\Delta T$. Hence, the exploration of the FES can be limited to the physically interesting regions of s
- Longer simulation time results in improved statistical accuracy in the relevant regions
- The risk of overfilling is avoided, and optimal use is made of the computer time
- Deciding when to stop the run is now simple, and post-processing is not necessary

WTMD converges to the exact free energy in the long time limit

Converging the FES

Recrossing

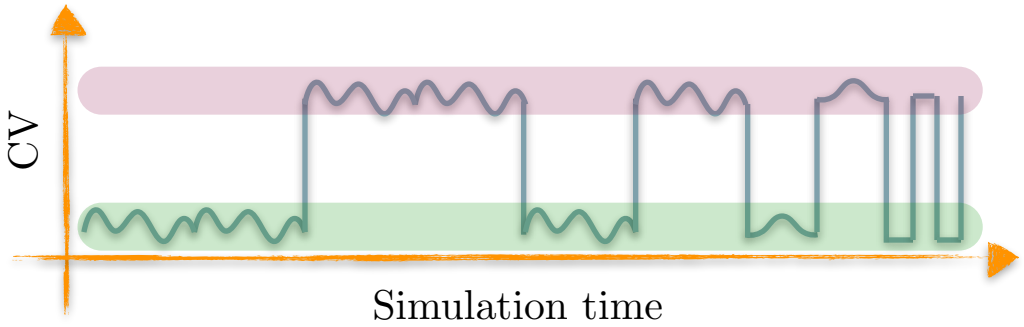


In order to converge the FES, crossing the free energy barrier just once is -definitely- not enough

One needs to observe several recrossings, back and forth

Till you actually fill the FES

In principle, when the FES is converged, the CV displays a *diffusive* behaviour



This is not easily observed, but even in the ideal case, you need time to converge

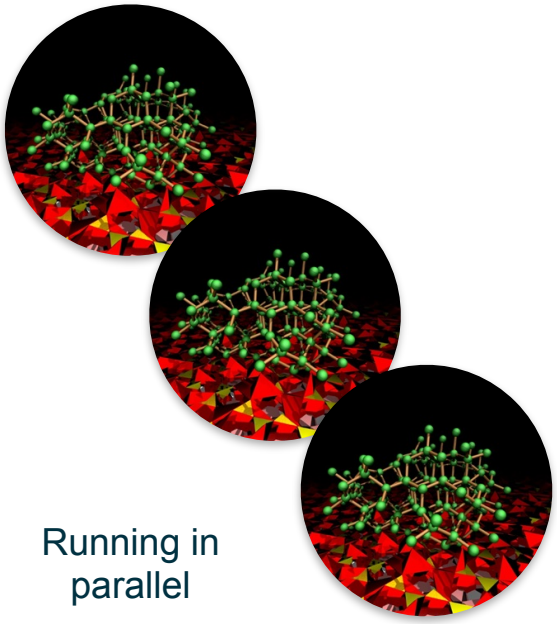
Multiple Walkers

Speeding up things

Especially within ab initio MD, computational time is an issue
What if if takes to long to even cross the barrier once?

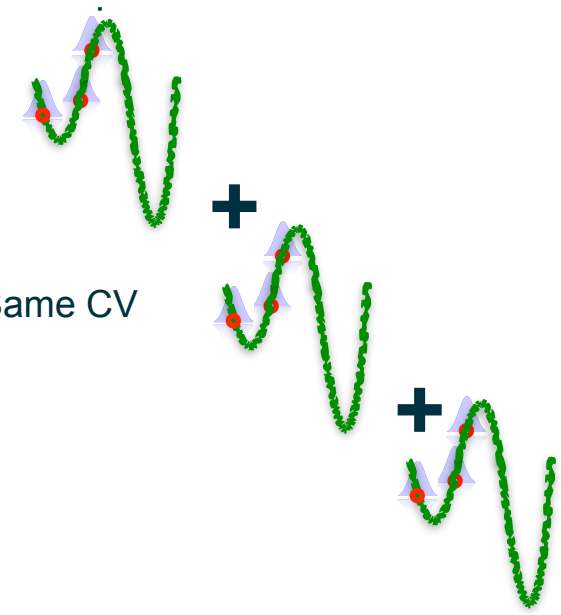
Multiple Walkers Metadynamics:
The simplest way to parallelise a Metadynamics run

Multiple replicas



Running in parallel

Same CV



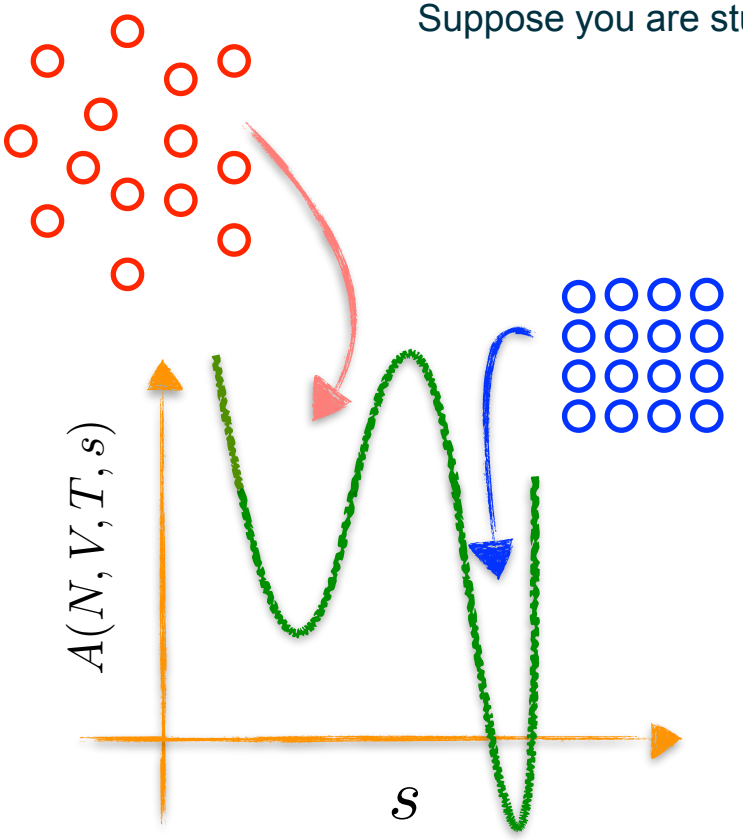
Much faster!

The deposited bias is shared among the different replicas so that the history dependent bias depends on the whole history

Adaptive Gaussian

Avoiding traps

Suppose you are studying crystallization from the melt



The liquid basin is - in whatever CV you choose - typically much wider than the crystalline basin

The width of the Gaussian we deposit is always the same!

Inefficient sampling
And, you can easily get stuck

Adaptive Gaussians: we change the width of the Gaussians on the fly

Order parameters

The biggest issue of them all

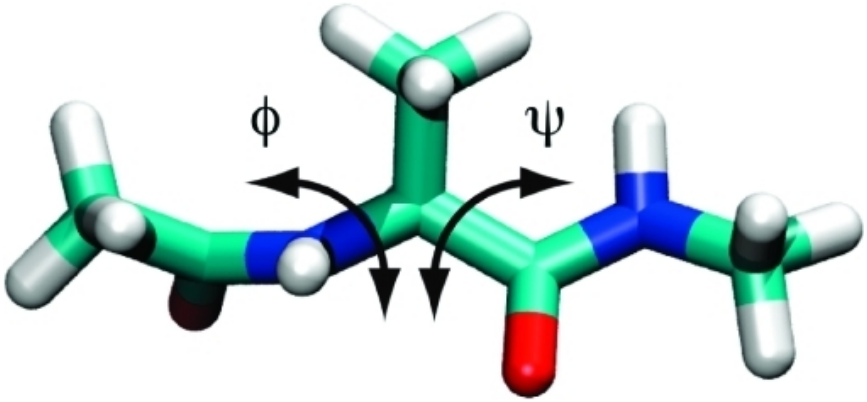
The choice of the order parameter is the most important thing to take care of
in ***every*** enhanced sampling method

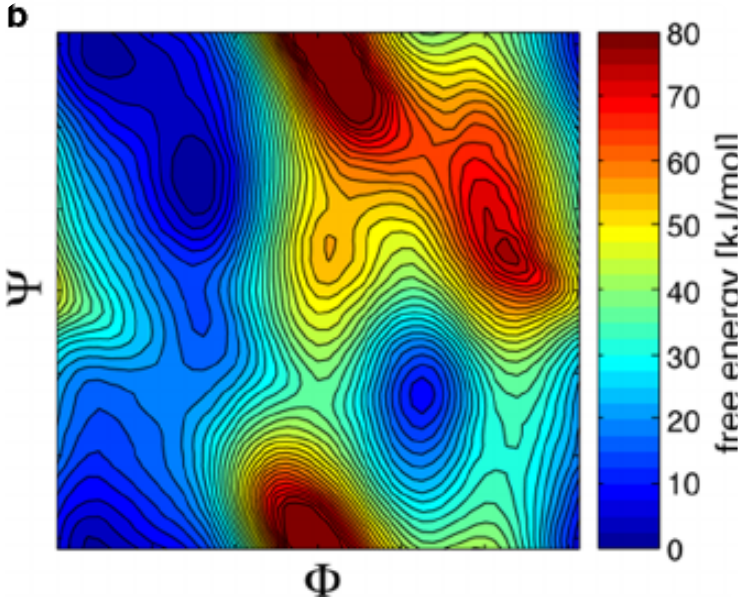
For one thing, the order parameter defines the FES you are going to get

And thus, the description of the process you are studying

Then,
the ***more complex*** the rare event you are interested in,
the ***more difficult*** is to choose the right CV. Or CVs

An easy textbook example: alanine dipeptide

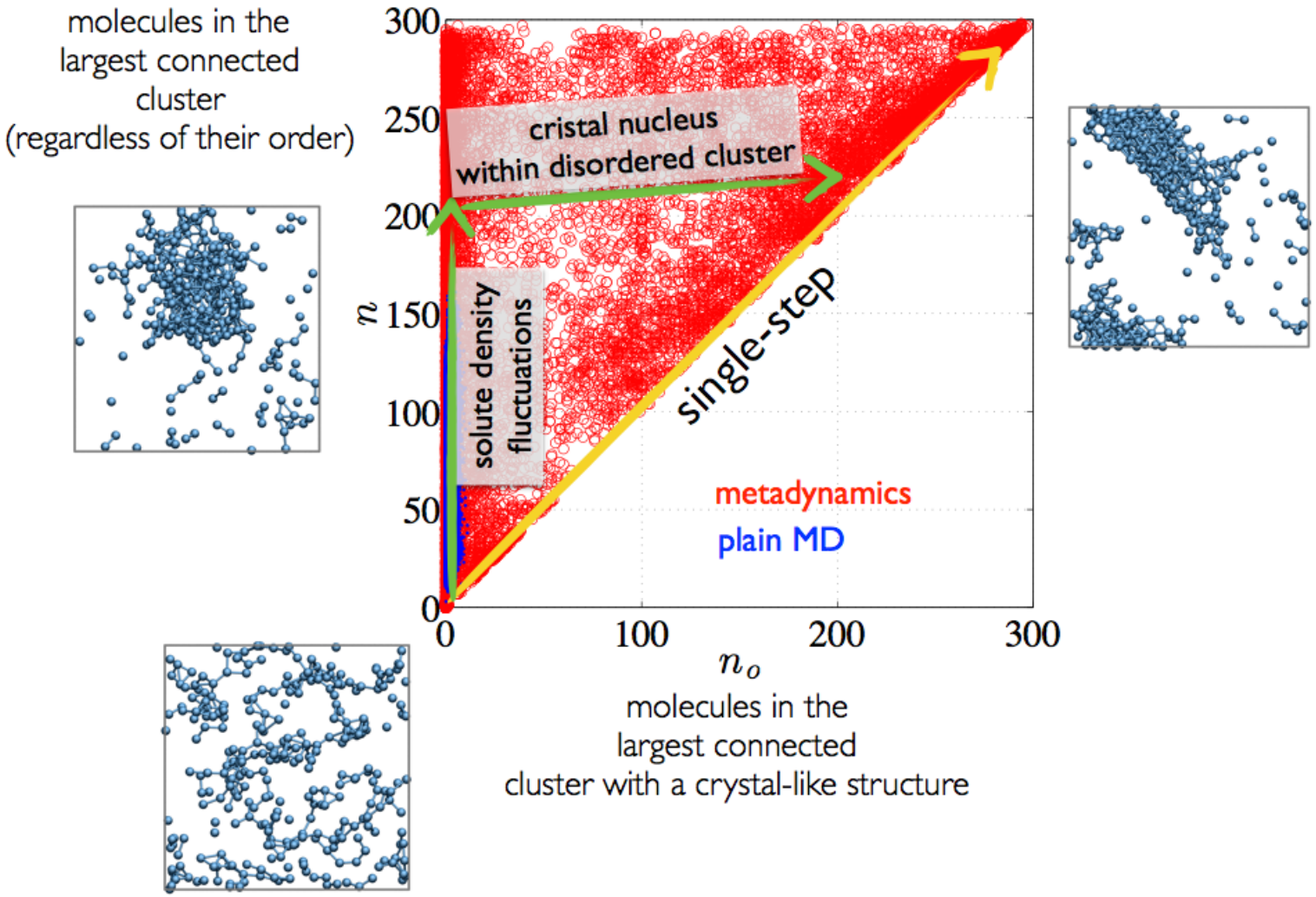




Order parameters

The biggest issue of them all

A complex example [courtesy of Matteo Salvalaglio (Chem. Eng. @ UCL)]:
crystal nucleation of molecules in solution



Order parameters

The biggest issue of them all

Common issues with order parameters:

- They miss something, something you need to describe properly the process you are interested in
- They do not distinguish well enough the different basins of the FES
- Degenerate order parameters: one value of the CV corresponds to (too many) states of the system

How do you know whether you have chosen a proper CV or not?

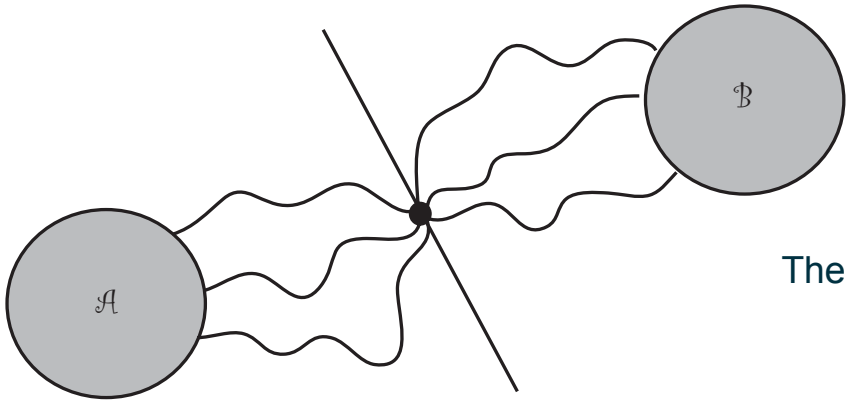
Sometimes the method simply won't work

Even if it does - in the sense that you get a FES - care must be taken

1. Verify the stability of the minima you have found
2. Perform a committor analysis

Committor Analysis

A very useful tool



$p_{\mathcal{B}} = 1/2$

Isocommittor surface

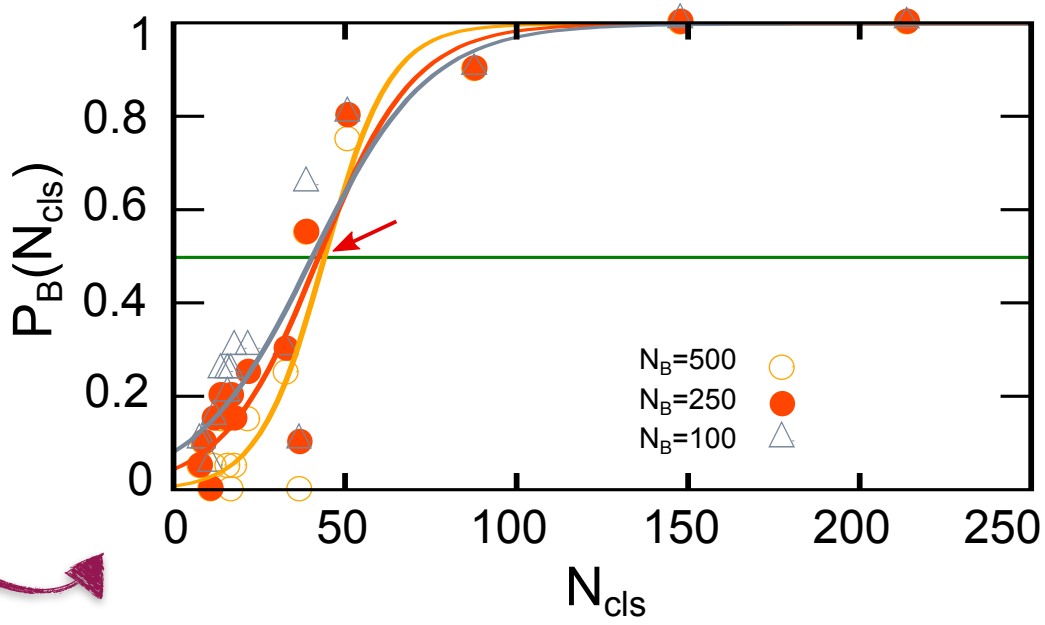
$$p_{\mathcal{B}}(\mathbf{r})$$

The probability that the system, starting from a given point along e.g. the A-B path, will end up in basin B

In practice:

Sets of *n* (many!) MD runs starting from different points

(for each run we randomise the initial velocity according to the Maxwell-Boltzmann at the temperature of interest)



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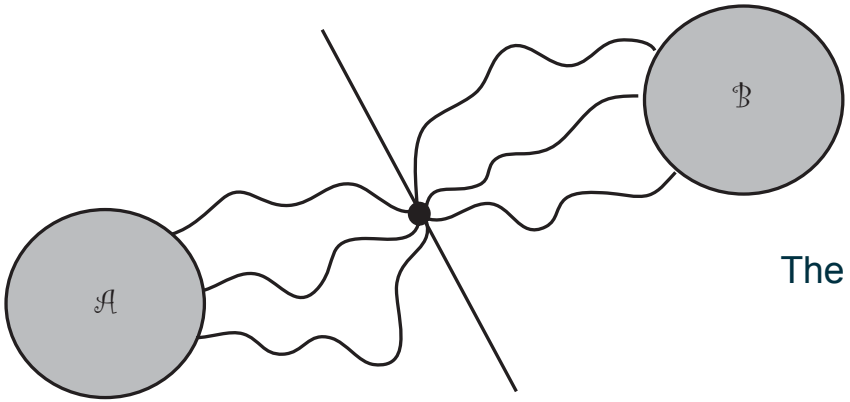

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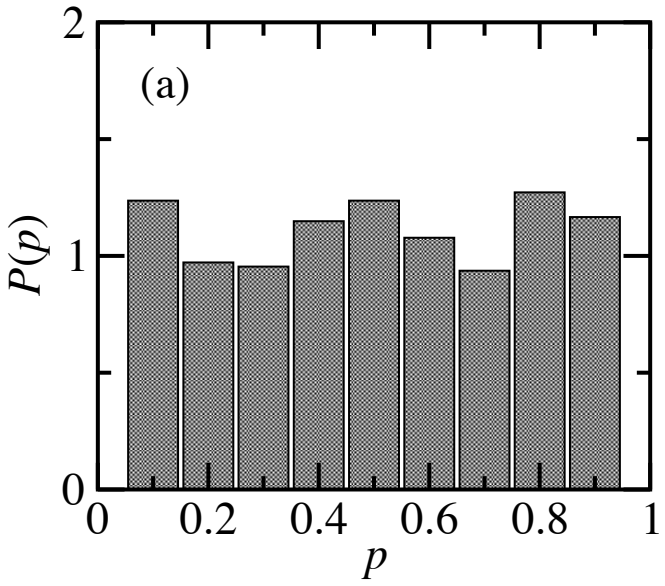

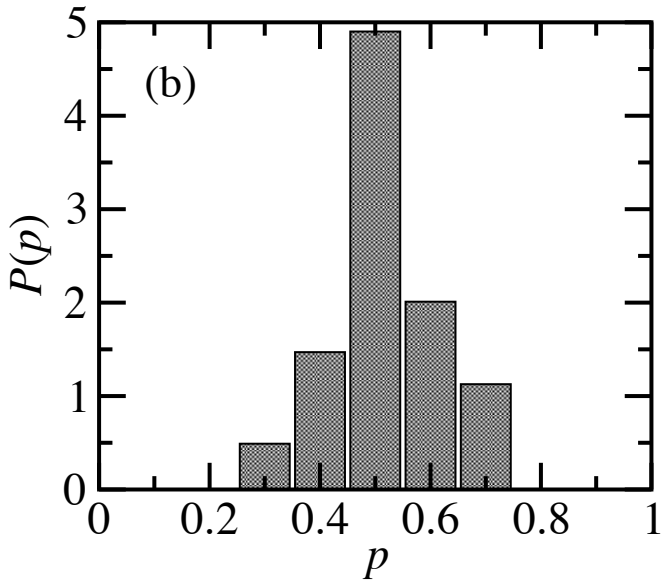

Which free energy method do I choose

Should be the system, it's your background

In principle:

The system itself and the rare event you are interested in should point you to the right approach

In practice:

It mainly depends on your very personal background

It's important to be willing to experiment!

Enhanced sampling methods very rarely work out of the box - and implementing them can be a pain

For instance: in metadynamics you need the derivatives of your CVs, as the forces due to the bias are a function of the derivative of the CVs with respect of the atomic positions

PLUMED

Metadynamics (and much more) ready to go
[on top of e.g. CP2K]

http://www.plumed.org

End of lesson

Something

- Learning Outcomes
  - Metadynamics: ***the concept of a history-dependent bias***
  - The key choice: ***why order parameters do matter***

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